CH14 – MATHEMATICAL REASONING

Exercise 14.1 Page No. 324

1. Which of the following sentences are statements? Give reasons for your answer.
(i) There are 35 days in a month.
(ii) Mathematics is difficult.
(iii) The sum of 5 and 7 is greater than 10.
(iv) The square of a number is an even number.
(v) The sides of a quadrilateral have equal lengths.
(vi) Answer this question.
(vii) The product of (–1) and 8 is 8.
(viii) The sum of all interior angles of a triangle is 180°.
(ix) Today is a windy day.
(x) All real numbers are complex numbers.

Solution:
(i) The maximum number of days in a month is 31, so this sentence is incorrect. Therefore, it is a statement.
(ii) This sentence is subjective. For some people, Mathematics can be easy, and for others, it can be difficult. Therefore, it is not a statement.
(iii) The sum of 5 and 7 is 12, and it is greater than 10. Therefore, this sentence is always correct. Hence, it is a statement.
(iv) This sentence can be sometimes correct and sometimes incorrect. For example, the square of 2 is an even number, but the square of 3 is an odd number. Hence, it is not a statement.
(v) This sentence can be sometimes correct and sometimes incorrect. For example, squares and rhombi have sides of equal lengths, whereas trapezia and rectangles have sides of unequal lengths. Therefore it is not a statement.
(vi) It is an order. Hence, it is not a statement.
(vii) The given sentence is incorrect because the product of (-1) and 8 is -8. Hence, it is a statement.
(viii) The given sentence is correct, and therefore, it is a statement.
(ix) The given sentence is not a statement because the day that is being referred to is not evident from the sentence.
(x) The given sentence is always correct because all real numbers can be written as $a \times 1 + 0 \times i$. Hence, it is a statement.

2. Give three examples of sentences which are not statements. Give reasons for the answers.
Solution:

The three examples of sentences which are not statements are given below:
(i) He is a doctor.
In the given sentence, it is not evident to whom 'he' is referred to. Hence, it is not a statement.
(ii) Geometry is difficult.
For some people, geometry can be easy, and for others, it can be difficult. Hence, this is not a statement.
(iii) Where is she going?
In this question, it is not evident to whom 'she' is referred to. Hence, it is not a statement.

Exercise 14.2 Page No. 329

1. Write the negation of the following statements:
(i) Chennai is the capital of Tamil Nadu.
(ii) $\sqrt{2}$ is not a complex number.
(iii) All triangles are not equilateral triangles.
(iv) The number 2 is greater than 7.
(v) Every natural number is an integer.
Solution:
(i) Chennai is not the capital of Tamil Nadu.
(ii) $\sqrt{2}$ is a complex number.
(iii) All triangles are equilateral triangles.
(iv) The number 2 is not greater than 7.
(v) Every natural number is not an integer.
2. Are the following pairs of statements negations of each other?
(i) The number x is not a rational number.
The number x is not an irrational number.
(ii) The number x is a rational number.
The number x is an irrational number.
Solution:
(i) The negation of the first statement is 'the number x is a rational number'.

This is the same as the second statement because if a number is not an irrational number, then the number is a rational number.
Hence, the given statements are negations of each other.
(ii) The negation of the first statement is 'the number x is not a rational number. This means that the number x is an irrational number which is the same as the second statement.
Hence, the given statements are negations of each other.

3. Find the component statements of the following compound statements and check whether they are true or false.
(i) Number 3 is prime, or it is odd.
(ii) All integers are positive or negative.
(iii) 100 is divisible by 3, 11 and 5.

Solution:
(i) The component statements are
(a) Number 3 is prime
(b) Number 3 is odd
Here, both statements are true.
(ii) The component statements are as follows:
(a) All integers are positive
(b) All integers are negative
Here, both statements are false.
(iii) The component statements are as follows:
(a) 100 is divisible by 3
(b) 100 is divisible by 11
(c) 100 is divisible by 5
Here, the statements (a) and (b) are false, and (c) is true.

Exercise 14.3 Page No. 334

1. For each of the following compound statements, first identify the connecting words and then break them into component statements.
(i) All rational numbers are real and all real numbers are not complex.
(ii) Square of an integer is positive or negative.
(iii) The sand heats up quickly in the Sun and does not cool down fast at night.
(iv) $x = 2$ and $x = 3$ are the roots of the equation $3x^2 - x - 10 = 0$.

Solution:

(i) In this sentence, 'and' is the connecting word
The component statements are as follows:
(a) All rational numbers are real
(b) All real numbers are not complex
(ii) In this sentence, 'or' is the connecting word
The component statements are as follows:
(a) Square of an integer is positive
(b) Square of an integer is negative
(iii) In this sentence, 'and' is the connecting word
The component statements are as follows:
(a) The sand heats up quickly in the Sun
(b) The sand does not cool down fast at night
(iv) In this sentence, 'and' is the connecting word
The component statements are as follows:
(a) $x = 2$ is the root of the equation $3x^2 - x - 10 = 0$
(b) $x = 3$ is the root of the equation $3x^2 - x - 10 = 0$

2. Identify the quantifier in the following statements and write the negation of the statements.
(i) There exists a number which is equal to its square.
(ii) For every real number x, x is less than $x + 1$.
(iii) There exists a capital for every state in India.
Solution:
(i) Here, the quantifier is 'there exists'.
The negation of this statement is as follows:
There does not exist a number which is equal to its square.
(ii) Here, the quantifier is 'for every'.
The negation of this statement is as follows:
There exist a real number x, such that x is not less than $x + 1$.
(iii) Here, the quantifier is 'there exists'.
The negation of this statement is as follows:
In India, there exists a state which does not have a capital.

3. Check whether the following pair of statements is a negation of each other. Give reasons for the answer.
(i) $x + y = y + x$ is true for every real number x and y.
(ii) There exists real numbers x and y for which $x + y = y + x$.
Solution:
The negation of statement (i) is as given below:

There exist real numbers x and y for which $x + y \neq y + x$
Now, this statement is not the same as statement (ii).
Hence, the given statements are not a negation of each other.

4. State whether the "Or" used in the following statements is "exclusive "or" inclusive. Give reasons for your answer.
(i) Sun rises or Moon sets.
(ii) To apply for a driving licence, you should have a ration card or a passport.
(iii) All integers are positive or negative.

Solution:
(i) It is not possible for the Sun to rise and the Moon to set together. Hence, the 'or' in the given statement is exclusive.
(ii) Since a person can have both a ration card and a passport to apply for a driving license. Hence, the 'or' in the given statement is inclusive.
(iii) Since all integers cannot be both positive and negative. Hence, the 'or' in the given statement is exclusive.

Exercise 14.4 Page No. 338

1. Rewrite the following statement with "if-then" in five different ways conveying the same meaning.
If a natural number is odd, then its square is also odd.

Solution:
The five different ways of the given statement can be written as follows:
(i) A natural number is odd, indicating that its square is odd.
(ii) A natural number is odd only if its square is odd.
(iii) For a natural number to be odd, it is necessary that its square is odd.
(iv) It is sufficient that the number is odd for the square of a natural number to be odd.
(v) If the square of a natural number is not odd, then the natural number is not odd.

2. Write the contrapositive and converse of the following statements.
(i) If x is a prime number, then x is odd.
(ii) If the two lines are parallel, then they do not intersect in the same plane.
(iii) Something that is cold implies that it has a low temperature.
(iv) You cannot comprehend geometry if you do not know how to reason deductively.

(v) x is an even number implies that x is divisible by 4

Solution:

(i) The contrapositive of the given statement is as follows:
If a number x is not odd, then x is not a prime number.
The converse of the given statement is as follows:
If a number x is odd, then it is a prime number.

(ii) The contrapositive of the given statement is as follows:
If two lines intersect in the same plane, then the two lines are not parallel.
The converse of the given statement is as follows:
If two lines do not intersect in the same plane, then they are parallel.

(iii) The contrapositive of the given statement is as follows:
If something does not have a low temperature, then it is not cold.
The converse of the given statement is as follows:
If something is at a low temperature, then it is cold.

(iv) The contrapositive of the given statement is as follows:
If you know how to reason deductively, then you can comprehend geometry.
The converse of the given statement is as follows:
If you do not know how to reason deductively, then you cannot comprehend geometry.

(v) The given statement can be written as 'if x is an even number, then x is divisible by 4'.
The contrapositive of the given statement is as follows:
If x is not divisible by 4, then x is not an even number.
The converse of the given statement is as follows:
If x is divisible by 4, then x is an even number.

3. Write each of the following statements in the form "if-then".

(i) You get a job implies that your credentials are good.
(ii) The Banana trees will bloom if it stays warm for a month.
(iii) A quadrilateral is a parallelogram if its diagonals bisect each other.
(iv) To get A^+ in the class, it is necessary that you do the exercises in the book.

Solution:

(i) If you get a job, then your credentials are good.
(ii) If the Banana trees stay warm for a month, then the trees will bloom.
(iii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

(iv) If you want to score an A^+ in the class, then you do all the exercises in the book.

4. Given statements in (a) and (b). Identify the statements given below as contrapositive or converse of each other.

(a) If you live in Delhi, then you have winter clothes.

(i) If you do not have winter clothes, then you do not live in Delhi.

(ii) If you have winter clothes, then you live in Delhi.

(b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram.

(ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

Solution:

(a) If you live in Delhi, then you have winter clothes.

(i) If you do not have winter clothes, then you do not live in Delhi [Contrapositive of statement (a)].

(ii) If you have winter clothes, then you live in Delhi [Converse of statement (a)]

(b) If a quadrilateral is a parallelogram, then its diagonals bisect each other.

(i) If the diagonals of a quadrilateral do not bisect each other, then the quadrilateral is not a parallelogram [Contrapositive of statement (b)].

(ii) If the diagonals of a quadrilateral bisect each other, then it is a parallelogram [Converse of statement (b)].

Exercise 14.5 Page No. 342

1. Show that the statement

p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by

(i) direct method

(ii) method of contradiction

(iii) method of contrapositive

Solution:

Let p: 'If x is a real number such that $x^3 + 4x = 0$, then x is 0'

q: x is a real number such that $x^3 + 4x = 0$

r: x is 0

(i) We assume that q is true to show that statement p is true and then show that r is true.

Therefore, let statement q be true

Hence, $x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

Since x is real, it is 0.

So, statement r is true.

Hence, the given statement is true.

(ii) By contradiction, to show statement p to be true, we assume that p is not true.

Let x be a real number such that $x^3 + 4x = 0$ and let $x \neq 0$

Hence, $x^3 + 4x = 0$

$x(x^2 + 4) = 0$

$x = 0$ or $x^2 + 4 = 0$

$x = 0$ or $x^2 = -4$

However, x is real. Hence, $x = 0$, which is a contradiction since we have assumed that $x \neq 0$.

Therefore, the given statement p is true.

(iii) By the contrapositive method, to prove statement p to be true, we assume that r is false and prove that q must be false.

$\sim r: x \neq 0$

Clearly, it can be seen that

$(x^2 + 4)$ will always be positive

$x \neq 0$ implies that the product of any positive real number with x is not zero.

Now, consider the product of x with $(x^2 + 4)$

$\therefore x(x^2 + 4) \neq 0$

$x^3 + 4x \neq 0$

This shows that statement q is not true.

Hence, it proved that

$\sim r \Rightarrow \sim q$

Hence, the given statement p is true.

2. Show that the statement "For any real numbers a and b, $a^2 = b^2$ implies that $a = b$" is not true by giving a counter-example.

Solution:

The given statement can be written in the form of 'if then' is given below:

If a and b are real numbers such that $a^2 = b^2$, then $a = b$

Let p: a and b are real numbers such that $a^2 = b^2$

q: $a = b$

The given statement has to be proved false. To show this, two real numbers, a and b, with $a^2 = b^2$, are required such that $a \neq b$.
Let us consider a = 1 and b = – 1
$a^2 = (1)^2$
= 1 and
$b^2 = (-1)^2$
= 1
Hence, $a^2 = b^2$
However, $a \neq b$
Therefore, it can be concluded that the given statement is false.

3. Show that the following statement is true by the method of contrapositive.
***p*: If x is an integer and x^2 is even, then x is also even.**
Solution:
Let p: If x is an integer and x^2 is even, then x is also even
Let q: x be an integer and x^2 be even
r: x is even
By the contrapositive method, to prove that p is true, we assume that r is false and prove that q is also false
Let x is not even
To prove that q is false, it has to be proved that x is not an integer or x^2 is not even.
x is not even indicates that x^2 is also not even.
Hence, statement q is false.
Therefore, the given statement p is true.

4. By giving a counter example, show that the following statements are not true.
(i) *p*: If all the angles of a triangle are equal, then the triangle is an obtuse angled triangle.
(ii) *q*: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.
Solution:
(i) Let q: All the angles of a triangle are equal
r: The triangle is an obtuse angled triangle
The given statement p has to be proved false.
To show this, the required angles of a triangle should not be an obtuse angle.

We know that sum of all the angles of a triangle is 180^0. Therefore, if all three angles are equal, then each angle measures 60^0, which is not obtuse. In an equilateral triangle, all angles are equal. However, the triangle is not an obtuse-angled triangle.

Hence, it can be concluded that the given statement p is false.

(ii) The given statement is

q: The equation $x^2 - 1 = 0$ does not have a root lying between 0 and 2.

This statement has to be proven false

To show this, let us consider

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

One root of the equation $x^2 - 1 = 0$, i.e. the root $x = 1$, lies between 0 and 2

Therefore, the given statement is false.

5. Which of the following statements are true and which are false? In each case, give a valid reason for saying so.

(i) *p*: Each radius of a circle is a chord of the circle.

(ii) *q*: The centre of a circle bisects each chord of the circle.

(iii) *r*: Circle is a particular case of an ellipse.

(iv) *s*: If x and y are integers such that $x > y$, then $-x < -y$.

(v) *t*: $\sqrt{11}$ is a rational number.

Solution:

(i) The given statement p is false.

As per the definition of a chord, it should intersect the circle at two distinct points.

(ii) The given statement q is false.

The centre will not bisect that chord which is not the diameter of the circle. In other words, the centre of a circle only bisects the diameter, which is the chord of the circle.

(iii) The equation of an ellipse is,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

If we put a = b = 1, then, we get

$x^2 + y^2 = 1$, which is an equation of a circle

Hence, a circle is a particular case of an ellipse.

Therefore, statement r is true.

(iv) $x > y$

By the rule of inequality
$-x < -y$
Hence, the given statement s is true.
(v) 11 is a prime number
We know that the square root of any prime number is an irrational number.
Therefore $\sqrt{11}$ is an irrational number.
Hence, the given statement t is false.

Miscellaneous Exercise Page No. 345

1. Write the negation of the following statements:
(i) *p*: For every positive real number *x*, the number *x* – 1 is also positive.
(ii) *q*: All cats scratch.
(iii) *r*: For every real number *x*, either $x > 1$ or $x < 1$.
(iv) *s*: There exists a number *x* such that $0 < x < 1$.
Solution:
(i) The negation of statement p is given below:
There exists a positive real number x, such that x – 1 is not positive.
(ii) The negation of statement q is given below:
There exists a cat which does not scratch.
(iii) The negation of statement r is given below:
There exists a real number x, such that neither $x > 1$ nor $x < 1$.
(iv) The negation of statement s is given below:
There does not exist a number x, such that $0 < x < 1$.
2. State the converse and contrapositive of each of the following statements:
(i) *p*: A positive integer is prime only if it has no divisors other than 1 and itself.
(ii) *q*: I go to a beach whenever it is a sunny day.
(iii) *r*: If it is hot outside, then you feel thirsty.
Solution:
(i) Statement p can be written in the form 'if then' as follows:
If a positive integer is prime, then it has no divisors other than 1 and itself.
The converse of the statement is given below:
If a positive integer has no divisors other than 1 and itself, then it is prime.
The contrapositive of the statement is given below:
If a positive integer has divisors other than 1 and itself, then it is not prime.
(ii) The given statement can be written as follows:

If it is a sunny day, then I go to a beach.
The converse of the statement is given below:
If I go to a beach, then it is a sunny day.
The contrapositive of the statement is given below:
If I do not go to a beach, then it is not a sunny day.
(iii) The converse of statement r is given below
If you feel thirsty, then it is hot outside.
The contrapositive of statement r is given below:
If you do not feel thirsty, then it is not hot outside.

3. Write each of the statements in the form "if *p*, then *q*".
(i) *p*: It is necessary to have a password to log on to the server.
(ii) *q*: There is a traffic jam whenever it rains.
(iii) *r*: You can access the website only if you pay a subscription fee.

Solution:
(i) The statement p in the form 'if then' is as follows:
If you log on to the server, then you have a password.
(ii) The statement q in the form 'if then' is as follows:
If it rains, then there is a traffic jam.
(iii) The statement r in the form 'if then' is as follows:
If you can access the website, then you pay a subscription fee.

4. Rewrite each of the following statements in the form "*p* if and only if *q*".
(i) *p*: If you watch television, then your mind is free, and if your mind is free, then you watch television.
(ii) *q*: For you to get an A grade, it is necessary and sufficient that you do all the homework regularly.
(iii) *r*: If a quadrilateral is equiangular, then it is a rectangle, and if a quadrilateral is a rectangle, then it is equiangular.

Solution:
(i) You watch television if and only if your mind is free.
(ii) You get an A grade if and only if you do all the homework regularly.
(iii) A quadrilateral is equiangular if only if it is a rectangle.

5. Given below are two statements:
p: *25 is a multiple of 5.*
q: 25 is a multiple of 8.
Write the compound statements connecting these two statements with "And" and "Or". In both cases, check the validity of the compound statement.

Solution:

The compound statement with 'And' is as follows
25 is a multiple of 5 and 8
This is a false statement because 25 is not a multiple of 8.
The compound statement with 'Or' is as follows:
25 is a multiple of 5 or 8.
This is a true statement because 25 is not a multiple of 8, but it is a multiple of 5.

6. Check the validity of the statements given below by the method given against them.
(i) *p*: The sum of an irrational number and a rational number is irrational (by contradiction method).
(ii) *q*: If n is a real number with $n > 3$, then $n^2 > 9$ (by contradiction method).

Solution:
(i) The given statement is as follows:
p: The sum of an irrational number and a rational number is irrational.
Let us assume that the statement p is false. That is,
The sum of an irrational number and a rational number is rational.
Hence,
$\sqrt{a} + \frac{b}{c} = \frac{d}{e}$ where $\sqrt{a}$ is irrational, and b, c, d, and e are integers.
$\therefore d / e - b / c = \sqrt{a}$
But here, $d / e - b / c$ is a rational number, and $\sqrt{a}$ is an irrational number.
This is a contradiction. Hence, our assumption is false.
∴ The sum of an irrational number and a rational number is rational.
Hence, the given statement is true.
(ii) The given statement q is as follows:
If n is a real number with $n > 3$, then $n^2 > 9$,
Let us assume that n is a real number with $n > 3$, but $n^2 > 9$ is not true,
i.e., $n^2 < 9$
So, $n > 3$ and n is a real number,
By squaring both sides, we get
$n^2 > (3)^2$
This implies that $n^2 > 9$, which is a contradiction since we have assumed that $n^2 < 9$,
Therefore, the given statement is true, i.e. if n is a real number with $n > 3$, then $n^2 > 9$.

7. Write the following statement in five different ways, conveying the same meaning.

p: If a triangle is equiangular, then it is an obtuse-angled triangle.

Solution:

The given statement can be written in five different ways, as given below:

(i) A triangle is equiangular implies that it is an obtuse-angled triangle.

(ii) A triangle is equiangular only if the triangle is an obtuse-angled triangle.

(iii) For a triangle to be equiangular, it is necessary that the triangle is an obtuse-angled triangle.

(iv) For a triangle to be an obtuse-angled triangle, it is sufficient that the triangle is equiangular.

(v) If a triangle is not an obtuse-angled triangle, then the triangle is not equiangular.

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